Math 5870—Act 4

Sylvester Mensah

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Mathematical Explanation of β_1 in some different scenarios

Let's first consider a SLR model where all assumptions are met;

$$Y = \beta_0 + \beta_1 x + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma^2)$$

$$y_x = b_0 + b_1 x$$

In the case where there is a unit increase in x;

$$y_{x+1} = b_0 + b_1(x + 1)$$

Measuring the corresponding amount of change in y when for a unit increase in x;

$$y_{x+1} - y_x = b_1$$

What do we do when these assumptions are violated?

1)

Transformation for a model that violates the linearity assumption(between x and y) or big x values.

$$Y = \beta_0 + \beta_1 log(x) + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma^2)$$

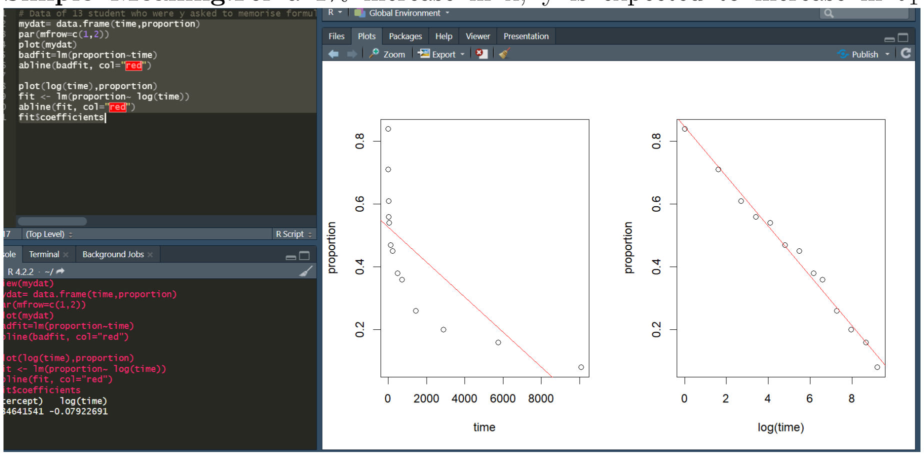
$$y_x = b_0 + b_1 log(x)$$

$$y_{x+1} = b_0 + b_1 log(x + 1) + \epsilon$$

$$y_{x+1} - y_x = b_1 log(\frac{x + 1}{x})$$

$$y_j - y_i = log(\frac{x + 1}{x})^{b_1}$$

Simple Meaning: For a 1% increase in x, y is expected to increase in b_1



2)

Transformation for a model that violates normality and homoscedasticity or big Y values .

$$log(Y) = \beta_0 + \beta_1 x + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma^2)$$

$$log(y_x) = b_0 + b_1 x$$

$$log(y_{x+1}) = b_0 + b_1(x + 1)$$

$$log(\frac{y_{x+1}}{y_x}) = b_1$$

Therefore,

$$\frac{y_{x+1}}{y_x} = e^{b_1}$$

Simple Meaning: For a unit increase in x, log(y) is expected to increase by b_1

3)

Transformation for a model that violates normality, homoscedasticity and linearity or big y and x values .

$$log(Y) = \beta_0 + \beta_1 log(x) + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma^2)$$

$$log(y_x) = b_0 + b_1 log(x)$$

$$log(y_{x+1}) = b_0 + b_1 log(x + 1)$$

$$log(\frac{y_{x+1}}{y_x}) = b_1 log(\frac{x+1}{x})$$

$$\frac{y_{x+1}}{y_x} = (\frac{x+1}{x})^{b_1}$$

Simple Meaning:1% increase in x will cause a b_1 increase in $log(y)$